THE PROJECT

Lifelong Learning Programme

MiMa

Mathematics in the Making

MiMa – Mathematics in the Making

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Title
MiMa - Mathematics in the Making – The Project

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Preface

The European Union (EU) performs a key role in supporting and integrating the work in education and training of all the member states with the aim of improving and updating the states' systems. While respecting the diversity of national laws and individual state's priorities, the EU encourages collaboration among its member states by making available significant funds for the creation of innovative training courses, materials, practices and teaching methods in order to enhance learning.

The collaboration at EU level allows the countries to work together and to learn from each other through the exchange of good practice and through the development of a variety of innovations that increase the European dimension in education and training. The EU projects, in addition to contributing to the solution of important social, economic and environmental problems, also enhance a sense of European identity and promote the values of European citizenship amongst its citizens; this in turn supports the creation of social and cultural cohesion.

In this context, supported by the Comenius Multilateral Lifelong Learning Programme, the MiMa *experience intends to contribute, through theoretical considerations and concrete products, to the process of improving the quality of teaching and learning in the classroom with reference to the specific, sensitive and complex subject of mathematics.*

This publication represents a summary of the work carried out by the MiMa *partnership during the two year long project and collects together contributions from all the project partners. Following a short introduction and an explanation of the main project phases, the publication details the* MiMa *methodology and the main concepts behind it. A brief description of the primary education system in the five partner countries follows. The* MiMa *methodology, teacher training course and learning materials were tested in each of the partner countries. Two important chapters report the project outcomes through photographs of the laboratories (workshops) and exhibitions carried out.*

A heartfelt thank you to all those who participated in the realisation of this project, in particular to our partners who believed in the project and worked with commitment and dedication, overcoming the inevitable difficulties, to the achievement of the project goals.

Fabiana Mariani
MiMa Managing coordinator
P3 Poliedra Progetti in Partenariato

Introduction: The *MiMa* Project

Background

The idea for the project *Mathematics in the Making* (MiMa) was born out of the practical experience of common problems related to mathematics teaching in schools. It was known that the negative attitude of the majority of young students toward mathematics starts in the early stages of education. In addition, some European education systems register persistent under-performance in mathematics by their 15-years-old students in the *Program for International Student Assessment* (PISA) of the Organisation for Economic Co-operation and Development (OECD). Moreover, besides numerical competences, young students also lack fluency in verbal expression in their speaking and writing of concepts and the chains of thoughts which follow a logical pattern.

Students tend to perceive mathematics as a topic that is too abstract to be fully understood, requiring only repetitive and mechanical operations that have nothing to do with reality. Moreover, students – and parents too - think of mathematics as a very difficult topic that only naturally "gifted" people can comprehend. Indeed, the reasons given for the high percentage of students in some nations doing poorly in the mathematics section of the PISA test in 2009 (OECD 2009) were closely connected to these ideas: students' low capability in critical thinking accompanied by a tendency to mechanically apply mathematics rules; a huge difficulty in connecting mathematical competences with problem solving in real life situations; an inability to conceptualize and organize situations in daily life through mathematical processes; and a very low capacity to write down the logical explanation of a problem when called upon to do so. In addition, the emotional aspect of learning mathematics plays an important role. Children's first experiences are crucial, but students are too often anxious about mathematics; and some distort their learning choices in order to avoid it.

In this challenging context, the *MiMa* project was created by a consortium of partners, including four universities in Hungary, Italy, Portugal and the UK which have relevant experience in hands-on teaching methodologies and mathematics teacher education, and the German Mathematikum, a museum with a vast experience of developing, designing and constructing interactive mathematical exhibits. All the partners of the project work with

mathematics students at different stages of education, and they believe that working together at a European level, creating tools that gather the best practices of all them, can contribute hugely to the achievement of the common goal of preparing better skilled European students and citizens, more competitive in the market and more active in the society.

It was really impressive listening to the pupils.
(Head of Teacher Education
at Sheffield Hallam University, England)

The aim of the project is to help primary school students develop a stronger interest and competence in mathematics and to enhance their social and civic skills, so contributing to an increase in students' chances of success, both in school and in life. Indeed, feeling capable of understanding a subject generally considered very difficult will produce, besides higher attainment in tests, a higher self-confidence in one's own learning potentialities and thus an increased possibility of continuing to study mathematics in the future.

Having these purposes, the *MiMa* project has built an example of an innovative approach that can 'improve attitudes, raise attainment levels, and open up new learning possibilities' (Commission of the European Communities 2008, para 2.8).

The *MiMa* methodology and activities

The first step in our work was to build a new teaching methodology based on a concrete approach to mathematics, creating tools that gathered the best practices and experiences of all the partner countries. The partners together chose ten hands-on activities and developed a mathematical teaching approach for each one to include various possibilities for the children to make outputs that could be shown in a public exhibition. To support the teachers' use of these proposals, a rich and complete *Toolkit* has been prepared (available in English, Italian, Hungarian, Portuguese and German at http://www.mathematicsinthemaking.eu/) with all the information, templates and other materials needed to allow teachers and children to replicate the objects and activities contained in the proposals, and to explore the mathematical arguments and related multidisciplinary links; moreover, the *MiMa* partners made a short video for each activity, showing and explaining some aspects of each proposal.

The activity *Experiments with dice* introduces the children to the first ideas about probability, and lets them familiarize themselves with the meaning of the terms probable and improbable in relation to the results obtained by throwing regular, loaded or special dice.

In the activity *Logic Mazes* the children build their own mazes, imagining and defining the rules to follow to find the correct path.

The activities *Many colourful triangles* and *Frieze patterns* allow the children to explore some aspects of planar geometry. *Many colourful triangles* uses equilateral triangles to be connected in various ways to obtain new figures, such as a rhombus, trapezium, parallelogram and regular hexagon. Also, the children can modify the triangles to obtain beautiful Escher-like tessellations of the plane. *Frieze patterns* aims to deepen the comprehension of the planar isometries (translation, rotation, reflection and glide reflection): the children learn how to use them to make beautiful friezes starting from a given motif and applying planar isometries regularly over and over again.

A mathematical football works with polyhedra, starting from the Platonic ones and moving to the complex structure of a football, a truncated icosahedron, that the children can make in cardboard. In the activity *Mathematics of the beehive* the geometric properties of the beehive become clear by working to make colourful cardboard models of cubes obtained from six pyramids, of rhombic dodecahedra and of the bee cells. *Exploring cuboids* works on the relationship between 3-dimensional cuboids and their 2-dimensional representations; possible paths on the edges of a cuboid are explored with the use of a simple software game.

Some activities require working outdoors. *Making a mathematics trail* guides the children to observe the mathematical aspects - in nature, in architecture, in everyday objects around them - that can be found in a given location, for example, in the neighbourhood of the school. The children use their discoveries to organize a trail to engage others in sharing their observations. The activity *Solar System* guides the children in making a large outdoor model of the solar system; the main challenge of this activity is to scale correctly the dimensions of the sun and the planets and their mutual distances from each other. Finally, *Sundials* teaches children to understand how to use the shadows of the sun to tell the time, making and using various kinds of sundials.

Working with teachers

The second step has been to provide professional development opportunities for teachers to support them in organizing mathematics laboratories (or workshops) based on the *MiMa* activities. In these dedicated courses, through learning the guiding principles of the interactive and hands-on methodology applied in the laboratories and experiencing the *MiMa* activities for themselves, the teachers had the opportunity to enrich and enhance their teaching methodologies and to think about how to apply the *MiMa* principles with their own classes to increase the children's involvement in mathematics and help them develop more positive attitudes toward the subject.

Working with children

The third step has been for the teachers to run the mathematics laboratories in their classrooms with 8 to10 year old students, encouraging creativity, cooperation and group work between students, and with the clear aim of making the *MiMa* objects and activities to show in a public exhibition in which the children guide the visitors, explaining their exhibits to them. In this way, the children have to think about mathematics in a new way and prepare themselves to take responsibility for explaining the exhibits and their mathematical content. This enhances their self-confidence in their own learning potential and their capacity to effectively approach and understand mathematical concepts.

The *MiMa* exhibitions are the final step of the project. They have been organized in various ways in the partner countries. What has been in common between all the events is that the participating teachers have made a public exhibition, big or small according to the local possibilities, involving students in the organization of the event. Having studied the mathematical aspects related to the objects they have created, the children have become the mathematical guides for the visitors. In this way, the children had the opportunity to feel pride in their own mathematics, often in front of their parents and relatives, and to become protagonists themselves of the event. The direct involvement of the students in the organization and explanation to the public of the mathematical products they created is useful in further developing their overall competences, such as language skills, critical thinking and a sense of civic responsibility and engagement.

Reflections

In a natural way, the *MiMa* activities created local networks of participating teachers, cooperating and sharing their activities with enthusiasm. It is hoped that they will continue to be connected after the end of the project.

Some international contacts grew out of the project, with children corresponding across national boundaries, allowing them to start familiarizing themselves from a very early age with the idea of Europe and of working together with other European children in the same project. In addition, the *MiMa* videos, spoken in the various languages of the partners, but all equipped with simple English subtitles, gave an opportunity for children to try to decipher different languages, feeling comfortable in trying to use them to communicate.

Moreover, learning mathematics has gained a much greater visibility in our local communities by the *MiMa* exhibitions.

Thanks for inviting me to the event yesterday,
I really enjoyed it.
Please pass on my thanks too
to all the children who shared
their great work with me - they did a fantastic job!
(Head of the Centre for Education and Inclusion Research
at Sheffield Hallam University, England)

The common aspect of all the *MiMa* activities was enthusiasm: the teachers have been thrilled to find new ways to engage their children and the children have very much enjoyed studying and working to prepare the materials and to act as guides to the exhibitions. The visitors, especially the relatives of the children, were surprised by this innovative way of teaching, and interested and satisfied by seeing the children so seriously and happily involved in learning.

And we, the *MiMa* partners, are proud of and satisfied with the very positive feedback we have received from teachers, children, families and the public.

References

Commission of the European Communities (2008) *Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of the Regions - Improving competences for the 21st Century: an Agenda for European Cooperation on Schools* {SEC(2008) 2177} http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex:52008DC0425

Organisation for Economic Co-operation and Development (2009) PISA 2009 Key Findings http://www.oecd.org/pisa/keyfindings/pisa2009keyfindings.htm

Chapter one: Why *MiMa*?

From *hand* to *heart* to *head*

Children are curious by nature, interested in many things and they experiment and understand new things with pleasure. These abilities are outstandingly well supported by hands-on-activities which in turn promote real learning and the ownership of knowledge.

The approach to learning mathematics encapsulated in the *MiMa* methodology is from *hand* to *heart* to *head*, a way of conceptualising learning which first arose from the work of the Swiss educationalist Johann Heinrich Pestalozzi (1801). In this way of thinking about learning, knowledge is gained sustainably by providing first hands-on-activities in order to ground understanding. Such activity fosters joy and motivation, so that the new experiences enter the heart. In this way the new knowledge finally finds its way into the brain of the learning child. Learning from the senses was central to Pestalozzi who believed that such learning is morally sound and enriching and also useful and instructive for life. He valued group learning which is both self-paced and hands on, with children working together and supporting each other's learning. The *MiMa* methodology grows out of and supports such ways of working.

A model of cognition

How such an approach supports cognition is developed in the work of Jerome Bruner (1966). He advocates teaching in three steps which mirror the development of understanding: from an *enactive* level to an *iconic* level to a *symbolic* level. Hands-on-activities provide this first step of an enactive approach to new knowledge or new capabilities: children experience the concrete in order to understand. After sufficient practical experience, an iconic (pictorial, representational) level is possible which entails thinking about the experiences of the concrete and beginning to abstract from them. Children are able to use graphic or mental representations. Both of these stages are vital in the development of learning and are prerequisites for the final stage in which children can act strategically at the symbolic level and create, understand and use symbol systems. To deepen or scrutinize their knowledge, or to resolve uncertainties, children can go back to their first hands-on activities for help. These remain as metaphors in the mind to support symbolic thinking. The *MiMa* hands-on-activities enable children to comprehend mathematics

with their hands and to build reliable knowledge for contemporaneous and subsequent iconic and symbolic mathematical thinking.

In addition, learning with hands-on-activities helps the children to use and develop further their visual thinking. This is fundamental for the abstraction, for example, of three-dimensional object to two-dimensional drawings and symbols. The *MiMa* activities encourage children to learn to look at the world with a mathematical eye, questioning and discovering geometric and mathematical features in everything they encounter. This supports the meaning-making which is vital to the development of mathematical learning.

Learning together

Central to the *MiMa* methodology is the importance for learning of children working together. Sound principles for developing group work in primary classrooms (TRLP, 2005) are:

- emphasising sensitivity, trust, inclusion and mutual respect between the members of the group;
- making appropriate practical classroom arrangements that support group work;
- and providing activities that are group-worthy and which warrant talk.

These fit with the *MiMa* methodology and the activities provided in the *Toolkit*. Group work based on these principles opens up the space for dialogue and dialogic teaching. This is because the work is *collective*, with the children working on common tasks together; *reciprocal*, with the students listening to each other's ideas and exploring alternative points of view; and *supportive*, with the students feeling free to express their ideas without being worried about being "wrong" and working together to achieve shared understandings (Alexander, 2008).

As teachers well know, mistakes and misunderstandings are natural and can be used as a tool for letting children think and inquire about a subject. The children, during their work and discussion, can be encouraged to make questions and conjectures, right or wrong. The teacher can guide the children to correct the mistaken conjectures, for example, by experimenting with their incorrect ideas on the concrete object they are making. Moreover, testing ideas, whether they be right or wrong, is a way to introduce the children to the scientific method.

This collaborative learning does not preclude times of quiet reflection. These may easily occur when the children are engaged in the practical activities themselves or when they are deciding how to record their thinking or during the problem-solving phase itself. Group work may often involve an initial sharing of ideas about how to interpret a mathematical task or problem, followed by quiet independent thinking and then the coming together again of the group to share ideas.

We've got a full council meeting today so it's difficult to attend. I've nipped out between meetings to see this. I really wanted to come as I know the children and teachers have been working hard on it. It's really great. ***(Local councillor who is also a school governor for one of the schools in England)***

Lev Vygotsky's theories stress the fundamental role of social interaction in the development of cognition, stemming from social interactions from guided learning within the zone of proximal development as children and their partners co-construct knowledge (1962; 1978). The *MiMa* activities will provide rich opportunities for the teacher to support the children in learning together. The organization of the *MiMa* activities will offer extended time and space for the children to work together (including interacting with other people during the final exhibition), building in this way a social learning environment.

Explaining to others

Part of the *MiMa* methodology is that children will present and explain their work to others - other children, their parents and members of the wider community - through exhibitions of their work. A certain level of maturity in their understanding has to be reached before the students are able to present and explain what they achieved. By the end of the project activities, students should be able to explain their reasoning and how they solved a given problem. This is very important for the development of their learning. Some questions do not arise until the reasoning has to be explained to somebody else. Only then are they fully answered and understood.

Hands-on-activities offer children a way to learn how to solve problems on a very practical level, which can be the basis for reflection on the strategies one

has used. Experience shows that hands-on experiments are accompanied by a lot of communication, which provides a basis for metacognition. Through reflecting on their own learning process and the strategies they used, children get to know possible ways to solve a mathematical problem, which they can transfer to future learning situations. Such metacognitive experiences can help to restructure one's own thinking and support the understanding of new content being learned.

Democratic learning

The construction of knowledge itself is not merely cognitive and individual but inherently formed by the way it is acquired. Different pedagogies not only allow learners to acquire more or less knowledge in more or less effective ways; they also shape the nature of the knowledge produced (Povey & Burton, 2004). With democratic learning - an active, engaged, meaning-making approach - the knowledge acquired is shared, open and consensual.

The new National curriculum is so narrow and so focused on number. The MiMa activities are a great opportunity to show the children there's so much more to mathematics.
(English teacher)

The *MiMa* methodology supports democratic learning in the mathematics classroom. Practical hands on activities carried out in groups of learners encourage the children's creativity: they are both making things and making knowledge. This in turn encourages them not just to see themselves as passively receiving knowledge but as actively making it their own. By this, they develop a sense of epistemological authority, building self-esteem and a sense of self-worth. This is not an experience routinely provided for lower attainers in school nor those from less advantaged socio-economic backgrounds. Following John Dewey (1949), *MiMa* supports active, experimental engagement in enquiry based learning with the children developing and organising the activities, being actors not spectators in their learning, which accords with a democratic way of life.

Attainment for all

The *MiMa* methodology caters for all levels of current attainment. The activities are rich enough to warrant the attention of both those who have previously struggled with mathematics and those whose previous attainment has been high. Indeed, the development of the latter group will be particularly supported by the opportunities for creative problem-solving and challenge that the activities provide in contrast to more routine studies that they may have experienced previously.

References

Alexander, Robin. (2008). *Towards Dialogic Teaching: Rethinking classroom talk* (4th ed.) York, Dialogos.

Bruner, Jerome Seymour. (1966). *Toward a Theory of Instruction*. Cambridge, Ma., Harvard University Press.

Dewey, John. (1949). *Democracy and education: An introduction to the philosophy of education*. New York, Free Press.

Pestalozzi, Johann. (1801/1894). *How Gertrude Teaches her Children.* London, Swann Sonnenschein.

Povey, Hilary and Burton, Leone with Angier, Corinne and Boylan, Mark. (2004) Learners as authors in the mathematics classroom. In L. Burton (ed) *Learning Mathematics, from Hierarchies to Networks*, 232-245, London, Falmer, 1999, reprinted in B. Allen and S. Johnston-Wilder (eds) *Mathematics Education: Exploring the Culture of Learning*. London, RoutledgeFalmer.

Teaching and Learning Research Programme. (2005). *Improving pupil group work in classrooms: A new approach to increasing engagement and learning in everyday classroom settings in key stages 1, 2 and 3* No. Research Briefing No11). London, Teaching and Learning Research Programme.

Vygotsky, Lev. (1962). *Thought and Language*. Cambridge, Ma., The MIT Press.

Vygotsky, Lev. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, Ma., Harvard University Press.

The children started to think in relations: '10 years. I was born in 2005, now it is 2015. This means, that I would have been on my way to the sun for my whole life!'
(German teacher)

You could go nowhere with the children without them saying 'look, this is a square' and 'look, this is symmetric'.
(German teacher)

At the end of year three, usually the children's performance curve drops down, because of the pressure of the recommendations for secondary schools. The MiMa-activity was great to motivate, distract and activate the children.
(German teacher)

Chapter two: Organization of primary mathematics education in the MiMa partners' countries

England

In England children start primary school at the age of 4 or 5 and are generally taught mathematics with their year group. Setting or grouping children on the basis of their attainment is common in English primary schools (Marks 2013). Children with special educational needs are mostly taught in mainstream schools, often supported or taught by teaching assistants. At the end of Key Stage 2 (age 11) children sit national tests in mathematics. These high-stakes tests can result in a restricted curriculum, with an emphasis on preparing children for the tests rather than developing mathematical skills and understanding (ACME 2014).

The statutory curriculum for mathematics for primary children (ages 5 - 11) in most state-funded schools is set out in the National Curriculum and assessment arrangements introduced through the 1988 Education Reform Act. These have undergone several revisions, most recently in 2013 (DfE 2013). Free schools and academies do not have to follow the curriculum although most do. Teachers are required to set high expectations for all children, recognise individuals' needs and remove barriers to attainment. The current National Curriculum also requires teachers to develop pupils' fluency, numeracy and mathematical reasoning through all curriculum subjects. The mathematics curriculum aims to ensure that pupils 'become fluent in the fundamentals of mathematics', are able to reason mathematically and solve problems. (DfE 2013, p99). The mathematics curriculum was supported by the National Strategies between 1997 and 2010; in mathematics this provided detailed guidance and professional development for teachers. Documentation, both statutory and non-statutory, together with an increase in accountability structures appear to have constrained teachers' freedom to work collaboratively to develop a curriculum appropriate for the children they teach (ACME 2006).

Ten-year-olds are taught number (number and place value, addition, subtraction, multiplication and division, fractions, including decimals and percentages) as they work with larger numbers, develop connections and solve increasingly complex problems; measurement, geometry (properties of shapes, position and direction), statistics.

Germany

Primary school in Germany starts at the age of six years and lasts four years. Subsequently the children go to a secondary school of the school type that meets their performance level.

Already, before attending school, many children acquire basic mathematical competences and concepts in a playful way and with the aid of didactical material in kindergarten. Although there are regional guidelines for pre-school education, there are no specific regulations about what children have to learn in kindergarten. This leads to the fact that the children's mathematical pre-knowledge (and this applies to other subjects as well) may be very divergent when starting school.

Traditionally, the main goal for primary mathematics education in Germany was to teach children calculation with numbers up to one million. The emphasis was on arithmetic and other topics were often given less attention. Over time, the focus changed and other topics like geometry were included in the curriculum.

Since 2005, the Standards for Education (Bildungsstandards im Fach Mathematik für den Primarbereich) have been implemented in German schools. They are guidelines for education from primary school to high school. The standards for primary school define the content and competences which the children should reach by the end of years 2 and 4. They provide an output-orientation, which was influenced by studies such as TIMSS and PISA.

By the end of their primary school years, the children should have general competences in communicating, arguing, illustrating, modelling and problem solving and substantial competences in the fields of numbers and operations, statistics and probability, space and shape, magnitude and measuring and patterns and structures. The Standards for Education, which apply in all sixteen counties in Germany, provide a good base for the children's development of competences. Nevertheless, each county's Ministry for Education and Cultural Affairs is in charge of the subjects to be studied and their content in their own counties, and individual schools have to develop a school curriculum based on the nationwide standards.

Hands-on-learning has been part of primary mathematics education for several decades and is still being developed. Starting from objects and experiencing learning through actions, enhancing their knowledge in a

playful way, the children are supported individually and met at their personal learning level by the teachers. It is not only basic arithmetic that is practised by a continuous use of hands-on-activities, but also in the other content areas various materials and experiments are used to support children in developing (basic) concepts and enable a deeper understanding. Furthermore the children's general competences in communicating and problem solving are developed by doing hands-on-activities.

Activities like that should be done much more often!
(German parent)

With its significant level of hands-on-activities in the lessons, the primary school is a model for secondary schools. Hardly any other school type in Germany is as developed or takes a holistic approach and learning through discovering as seriously. How close or otherwise theory and praxis are at particular schools is not easy to analyse; however academics, politicians and teachers are strongly interested in continuing to implement hands-on-activities in schools.

Hungary

Primary education in Hungary consists of two stages, real primary school and low secondary school. In primary school, children first receive education from age 6 to 10, with most children continuing in the same school from years 5 to 8, where the subjects are taught by specialized teachers. The majority of Hungarian schools are public, but there are a number of private and church-run schools as well.

A core curriculum is mandated by the Ministry of Education (Dobos, Ocskó and Vásárhelyi 2001). Schools may also specify their own curricula based on the core curriculum. Primary school subjects include mathematics, language, environmental studies (embracing both natural and social environments), singing, drawing, physical education and information technology, to which a few more subjects may be added such as music, foreign language or chess. Primary school classes usually have two teachers, one teaching mathematics and some other subjects depending on the circumstances, the other literacy and all other subjects. Mandatory class periods may take place either both in the morning and in the afternoon where children spend all day in school from

8am to 4pm (having lunch in school as well). In some schools however pupils have classes only in the morning, with lunch and afternoon sessions being optional. The choice depends on local capabilities and parental preferences.

The mathematics curriculum is uniform throughout the whole system of public education. It can be divided into five major areas: functions (analysis); numeracy (algebra, number theory, arithmetic); geometry; probability theory; and combinatorics: the entire curriculum is interwoven with logic and set theory. Traditionally, the primary school taught counting and measuring. This was replaced, as a result of the work of Tamás Varga, by mathematics in the 1970. Even though one of the influences on Varga had been the 'new maths', the new Hungarian methodology was also in stark contrast to the formal nature of new maths. Varga's mathematics is built on experience, games and experimentation. Since then, many changes have occurred. The basic concept remains, but the quality of implementation varies widely. It depends on the school, but even more on the teacher.

In grade 4 (age 10) children take a National Assessment of Basic Competencies in mathematics. The Act of National Public Education introduced a three-tier school inspection system in 2011.

Italy

In Italy children enter primary school at the age of six (with some flexibility: under some conditions they can start at five and a half). Primary schooling lasts five years. In years 2 and 5 primary children take national tests, managed by Invalsi (http://www.invalsi.it). There is increasing concern that the pressure on schools to perform well in these tests distorts the curriculum.

The framework for the planning of the curriculum by the schools is given by the Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione (National guidelines for the curriculum of the preschool and of the first cycle of education) (Ministry of Education 2012). They stress that mathematics develops the skills to find the close relationship between thinking and doing offering in this way the tools to understand the reality around us. In particular, they underline its usefulness for the scientific comprehension of the world, and to learn to correctly communicate and discuss. A particular stress is put on the fact that to teach mathematics (and science in general) it is fundamental to use a laboratorial approach; in this context, laboratory means both a physical location with tools and exhibits, but also the moment in which the children actively work, proposing their

hypothesis, checking their consequences, discussing and developing their own knowledge. Also, the possibility of teaching by letting children play is suggested.

Mathematics practice is recognised as solving problem in the broadest sense. A particular emphasis is placed on the need to develop an adequate perception of mathematics, avoiding seeing mathematics 'as a set of rules to memorize and to apply' (Ministry of Education 2012, p52); also, on the need to become aware of the usefulness of mathematical tools to act in the real world. Unfortunately, this is based on a recognition that a large number of Italian students do indeed get the wrong idea of mathematics from school, and do not feel themselves able to master it. The Ministry's approach is quite new for Italian schools, in which traditionally teachers were expected to explain at the blackboard, and children expected to listen and to take notes; so the process of moving to a laboratorial approach is something that cannot be immediate, but will take some time and effort.

The curriculum envisages children developing knowledge and skills about numbers and computations, figures in the plane and in space (and skills in using drawing and measuring tools), representation and analysis of data in tables and graphs, uncertainty, problem solving and analysis of possible solving strategies, reading and understanding mathematical texts.

Portugal

The primary education system in Portugal, is composed of four years: 1st year (6-7 years old), 2nd year (7-8 years old), 3rd year (8-9 years old), 4th year (9-10 years old). It is the first cycle of the basic education system. In order to ensure high-standards in the primary education system, the Education and Science General Inspection service is the public agency responsible for its control, auditing and evaluation. The agency proposes and cooperates in the development of measures that aim to improve the educational system performance and, on a smaller scale, to ensure pedagogical and administrative support to schools. Further, it ensures the participation in the inspection structures of European schools as the Portuguese Education and Science Ministry representative.

Regarding the official curriculum of mathematics that must be followed by all schools (private or state), the main purposes of mathematical teaching are three-fold: the structure of reasoning, the analysis of the natural world and the interpretation of society. The official Portuguese Mathematics

Curriculum states: "the apprehension and hierarchisation of mathematical concepts, the systematic study of their properties and a clear and precise line of argument play a major role in the reasoning structuring" (Programa e Metas Curriculares Matemática 2013). Regarding the world that surrounds us, "Mathematics is essential to the proper understanding of the world's phenomena" as the "proficiency in mathematical instruments will enable the study of the phenomena". Finally, the "mathematical method is the major instrument for the analysis and understanding of the functioning of society", so that the "teaching of mathematics contributes to the exercise of a plain, informed and responsible citizenship".

The children got very enthusiastic about the mathematical trail and couldn't stop inventing tasks for their fellow students. Some of them were very creative (catapult made from bark)!
(German teacher)

In order to achieve the previously mentioned purposes, the following fundamental performances were established for students: identify concepts and objects; know the results; recognize intuitively the veracity of results in concrete examples; and understand generalizations. These performances will contribute to the achievement of the objectives of acquiring knowledge of facts and procedures and of the construction and development of mathematical reasoning. They will improve students' use of appropriate mathematical communication (oral and written), enable them to solve problems drawn from different contexts and lead to a holistic conception of mathematics.

As for the content of the official curriculum of mathematics, the subjects covered in this cycle of education will be introduced gradually, first from an experimental and concrete basis and then moving to a more abstract conception. In numbers and operations students will have to master the four algorithms corresponding to the operations of sum, subtraction, multiplication and division and show proficiency in mental arithmetic. The study of fractions, introduced during the 3rd year, is a key-subject covered during this cycle of education. In geometry and measure, the basic concepts such as points, collinear points, directions, lines, semi-lines and line segments, parallelism and perpendicularity are presented so that more complex objects,

namely polygons, circumferences, solids and angles, may be covered. Angles and line segment measure are also addressed so that a relation between geometry and measure may be established. Finally, regarding the subject of data treatment and organization, different processes of data presentation are covered and basic set theory vocabulary is introduced.

Assessment of students is carried out on a regular basis by the students' own teachers throughout the first cycle, culminating in a national examination at the end of the fourth year.

References

ACME (2014) The maths curriculum http://www.acme-uk.org/media/20272/mathscurriculum.pdf

ACME (2006) Position Paper: Key issues for the future of primary mathematics learning and teaching http://www.acme-uk.org/media/6942/acme%20position%20paper%20on%20primary%20mathematics.pdf

DfE (2013) National Curriculum in England: Primary Curriculum https://www.gov.uk/government/collections/national-curriculum

Dobos, Ocskó & Vásárhelyi, 2001, Reference levels in School Mathematics Education in Europe, Presentation to the European Mathematical Society, March 2001. http://www.emis.de/data/projects/reference-levels/EMS_HUNGARY.pdf

Marks, R. (2013) "The Blue Table Means You Don't Have a Clue": The Persistence of Fixed-Ability Thinking and Practices in Primary Mathematics in English Schools, *FORUM*, 55 (1), pp. 31-44.

Ministry of Education, 2012, Indicazioni Nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione. http://www.indicazioninazionali.it/J/

Programa e Metas Curriculares Matemática, (2013) Direção Geral de Educação, Governo de Portugal, 2013, p. 2, Available at http://www.dge.mec.pt/sites/default/files/Basico/Metas/Matematica/programa_matematica_basico.pdf

I'd love to join your project.
I am convinced that the first step to learning for children is tangible action, and I'm interested in every initiative which sets out to reinforce this.
(Hungarian professor at a teacher education institute)

Unfortunately I cannot attend Thursday's meeting and now I cannot even be involved in the creation of an exhibition. But I'm writing because the "mathematical hike / walk" keeps coming back to me constantly.
(Hungarian teacher)

The hands-on activities attracted the attention of the students with difficulties, and gave them the possibility to understand the subject and to succeed.
(Italian teacher)

Chapter three: The MiMa Laboratories

Overview

Ninety *MiMa* laboratories (workshops) occurred across the five participant countries, involving approximately 40 schools and over 110 teachers with the participation of nearly 1600 students. During this phase of the project, all ten *MiMa* activities were implemented and some activities - Frieze patterns, Logic mazes, Making a maths trail, Many colourful triangles and Modelling the solar system - were trialled simultaneously in several different countries.

The *MiMa* laboratories were beneficial in many different ways to both teachers and children. The effect on motivation was particularly stressed by teachers, either regarding their own activity:

The MiMa Project was a breakthrough in my activity as a teacher because it enabled a wide range of materials and released possible clues for the exploitation of the curriculum through fun activities, where students create their own knowledge, leading them to develop their mathematical reasoning.

or the attitude of their students:

I was surprised by the interest and willingness to participate showed by all the students, especially by those that present the most learning difficulties in this area.

The images which follow show the students' enthusiasm and their commitment to the hands-on tasks which were common features throughout the project.

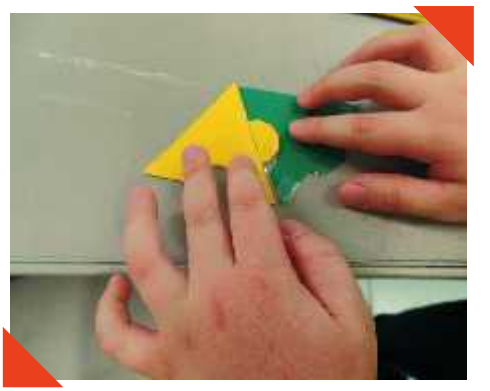
Experiments with dice

Experiments with dice allowed for many different games

Logic mazes

A giant logic maze.
The children had fun solving the maze.

Many colourful triangles



Exploring tilings

Frieze patterns

Constructing friezes

Mathematical football

Constructing a *MiMa* football

Mathematics of the beehive

Constructing the *MiMa* beehive - is it how the bees do it?

Exploring cuboids

Exploring the *Cuboids* software

Making a maths trail

Our maths trail visited many beautiful places.

Modelling the solar system

Building the sun

Sundials

Why and how does a sundial work?

Checking the time on a raised sundial

Chapter four: The *MiMa* exhibitions

Overview

A highlight of the *MiMa* project was the exhibitions which were held in the five partner countries. There was a great variety from country to country in the presentation of the children's results, that is, in the format of the exhibitions. There were some national exhibitions with all the schools together; some schools organized their own exhibitions; and sometimes both formats were realized independently. No matter how different the particular exhibitions were, they all had one thing in common: the children became experts and explained to the visitors how they had made their objects in class and the mathematics behind it.

This main goal of the *MiMa* project was realized in all partner countries: The children happily presented their results and talked enthusiastically about their experiences. Everybody could see how much work and consideration the teachers had put into the project to give their students the opportunity to present a beautiful exhibition. So it was no surprise that in addition to the children and their teachers, also many parents, siblings and friends came to visit the exhibitions to hear the children's explanations.

Here we give some impressions of the various exhibitions.

England

The presentation of a maths trail

Colourful mathematical footballs

Solving a logic puzzle

It was way more fun than working out of a textbook. You had to really think about what you were doing - or you would do it wrong. Also it was a good thing to do to get your brain ready and working before a lesson... Overall I thought it was a great idea and I recommend that you do it, or tell your teacher about it. (English student)

The children enjoyed the problem solving aspects of the logic mazes. They enjoyed coming up with their own problems trying to make them as complex as they could and then sharing those problems with other children. (English teacher)

Germany

Presentation of the planets of our solar system

Many colourful triangles

Student showing the beehive models

Marie is an intelligent and eloquent student but maths has always been her least favourite subject. During the activity she really fell for geometry. She even volunteered to present the activity. And she wasn't the only one. (German teacher)

It is fascinating to see what we learned as a family through the MiMa project. The universe and interplanetary distances were topic number one at the dinner table for days. (German mother)

Hungary

Last preparations
for the colourful triangles

Presenting
the various tilings

A real-life stepping stone maze

It was really instructive to watch the children at work as they sorted out problems (including organizational ones) and discussed ideas for solutions. At the exhibition they cleverly "evaluated" works and paid close attention to their task. (Hungarian teacher)

The kids asked me to keep the board game we had designed, and asked to play with it again. (Hungarian teacher)

Italy

The European flag made from bee cells

Isometries from the friezes activity

Children explaining their logic mazes

During the activity, I noticed with great pleasure that those children usually considered "shy" were able to master the situation and to explain the various phases of the work with self-confidence and with the correct vocabulary. (Italian teacher)

Portugal

Children marvelling at the mathematical footballs

Exploring cuboids – the children's lions

Demonstrating the various frieze operations

This project has become a more motivating way of working the curriculum content by giving the students the perspective of the real usefulness of mathematics in their daily lives. (Portuguese teacher)

Teacher, can we truncate other geometric solids and discover other figures? (Portuguese fourth grade student)

A child named Francesco was so fond of the tessellations and of the work of Escher that he decided to name himself Frescher.
(Italian teacher)

The students showed great pleasure in exploring geometry by building geometric figures and solids in Polydron pieces, especially a student with Asperger's syndrome. Normally, he doesn't show any interest in any kind of activity, nor does he socialize with his fellow students, but in this particular activity, he interacted with his peers.
(Portuguese teacher)

Chapter five: Future developments

We have many plans for the future for the continuation of the *MiMa* project, some already firmly planned and some currently more like dreams. We, the project partners, have gained enormously from our participation in the project and are fully committed to continuing our strong informal network - by group and individual email, through our open Facebook group and, we hope, by meeting face-to-face in each other's countries and at international mathematics education conferences. We have spoken about devising a new project that builds on the strengths of *MiMa* and the team working we have established, perhaps working with young learners from other age groups and linking with other mathematical themes, perhaps working with teachers to enhance and develop teachers' mathematics subject knowledge.

Many of our teachers too have made concrete plans for continuing *MiMa*. Some want to extend what they have learnt to other younger children. Some want to run *MiMa* teacher professional development sessions for neighbouring schools. Some want to work together to create more *MiMa*-style activities. Some want to make the MiMa exhibition a standard part of the school's annual calendar. What they all have in common is the intention to continue working with the *MiMa* methodology with their children. We are hoping that some of the partners may be able to tender for funds to support the rich range of extension activities at a national level.

Finally, and not least, the *MiMa* project website will continue to provide an open access archive for all the published project outputs. This will include the guidelines detailing the pedagogical principles behind the project; the teaching training course materials; the project newsletters and press releases relating to various stages of the project; the *Mima Toolkit* which contains details of the ten activities, the mathematics that the children will encounter and how to implement the activities in the classroom; practical support materials including both templates and guidance on the mathematics for teachers; ten video introductions to the activities; an electronic version of the Final Publication; and a variety of other materials. There will be links to the website from a variety of national fora and we hope very much in this way to keep the *MiMa* spirit alive.

One of the students showed great interest
in the Friezes activity, which was a very important challenge
for her because it allowed her to perform tasks
that helped her overcome her problem: dyslexia.
(Portuguese teacher)

Since we began the Colourful triangles activity
the students began to use
a more rigorous mathematical language.
(Portuguese teacher)

Funny! We are building beehives with maths.
(Portuguese student)

I hadn't noticed before
that there were so many symmetries around us.
I thought they were only in mirrors...
(Portuguese student)

Chapter six: The *MiMa* partners

Eotvos Lorand University, Hungary

Founded in 1635, ELTE is the largest university in Hungary.

The Centre of Mathematics Education is a part of the Mathematical Institute (led, a few years ago, by László Lovász). In addition to teacher training and other mathematics classes, we also conduct research. Research topics range from basic research in mathematics education to the coordination of computer work and hands-on activities, from improving the efficiency of learning in school of students from low families of low socio-economic status to the history of mathematics from a mathematics education perspective. The fruits of our research reach our students quickly as part of elective courses at first and are integrated into the regular curriculum later on.

Mathematikum, Germany

The Mathematikum is the first mathematical science centre worldwide, founded in 2002 by Professor Albrecht Beutelspacher. It is located in Giessen, a medium sized town in the middle of Germany.

The exhibition contains over 170 experiments which allow the visitors to explore fascinating mathematical effects. Visitors can try to solve puzzles, they may reflect themselves in different mirror experiments, they can build bridges and much more. There are experiments connected to all areas of mathematics and ones which also relate to subjects like physics, music, and arts.

The exhibition was amazing.
Even the smaller ones enjoyed our materials.
It was "wow"!
(Portuguese student)

These hands-on and minds-on activities open a new door to mathematics for many people. The main visitor groups are school classes (of every level) and families. But also professional mathematicians will find new and surprising things. In 2009 the Mini-Mathematikum was opened as a special section of Mathematikum. The cosy atmosphere invites 4 to 8 year old children to discover mathematics in their own way. Here, all experiments are designed to meet the needs of younger children.

Beside the exhibition, the Mathematikum is engaged in various projects for bringing mathematics to the general public, especially to children and young people.

In particular, the various travelling exhibitions are taking the new image of mathematics to many people.

P3 Poliedra Projects in Partnership, Italy

P3 Poliedra Projects in Partnership is the managing coordinator of the *MiMa* project. It was formally established in 2010 and is an association of professional consultants with many years experience in the following areas: European Project Planning/Design, Project and Financial Management, Project Cycle Management, Participatory Planning and Professional Training. All the P3 members are involved in the *MiMa* project; three of them are certified PMP® by the PMI Institute.

Sheffield Hallam University, England

The Mathematics Education Centre at Sheffield Hallam University has an international reputation in its field, and our team brings a wealth of experience in mathematics education to the wide range of activities we carry out. These include initial teacher education courses, the professional development of teachers, curriculum development projects, activities with pupils, teaching and supervising at undergraduate, Masters and doctoral level and research in mathematics education. Our aim is to enrich the learning of mathematics and to inspire the creative teaching of mathematics in everything we do.

The students' feedback was that it became easier to grasp the content and as such we will implement these activities in the coming school year, including the Friezes.
(Portuguese teacher)

Three of us from the Mathematics Education Centre are involved in the MiMa project: Dr Gill Adams whose particular interests lie in mathematics teacher professional learning; Colin Jackson who is currently researching social justice issues in mathematics education; and Professor Hilary Povey who researches equity issues in the learning of mathematics and who has substantial experience of curriculum innovation in mathematics. The fourth member of our team is a practising primary teacher, Jan Hedge.

Universidade Nova de Lisboa, Portugal

The Mathematics Department of the Science and Technology Faculty of the Universidade NOVA de Lisboa, beyond its didactical work, has a deep concern to open up to society, developing and supporting various outreach initiatives.

The *MiMa* team, Fátima Rodrigues, Gracinda Rita Guerreiro, Maria do Céu Soares, Nelson Chibeles Martins and Susana Baptista, are Assistant Professors in NOVA's mathematics department. In 2007, based on our experience as teachers and disseminators of mathematics, we created ClubeMath - a mathematics club for school students from 11 to 18 years old. In 2011, the club was extended admitting children from 6 to 18 years old. By presenting mathematics in a playful context through recreational and hands-on activities such as games, challenges and problem solving, ClubeMath aims to promote students' learning of mathematics and their enjoyment of doing so. We consider that the hands-on activities used at ClubeMath are crucial to the transfer of mathematical knowledge and can be applied to any level of education (including university level) allowing for an effective consolidation of knowledge.

With MiMa we learned more than just mathematics.
We learned to work as a team
and to always try to go farther.
(Portuguese class)

The team also organizes playful mathematical activities for families, who are the students' key influencers, and develops a variety of actions directed towards schools (for instance, receiving school students in NOVA and running courses for teachers). We are also involved in various outreach activities for the general community, such as the organization of conferences, mathematics exhibitions and the development of mathematical games.

My students said: Can we stay in the classroom working
and not go to the playground??
No more MiMa projects ?!
We could make more at lunchtime...
(Portuguese teacher)

Università degli Studi di Perugia, Italy

The Department of Mathematics and Informatics of the University of Perugia cooperates with the National Piano Lauree Scientifiche to spread new and involving approaches to the didactics of mathematics.

The core of these activities is the *Galleria di Matematica*, a permanent exhibition of mathematical objects, completely imagined and realized under the direction of Emanuela Ughi, in the Polo Museale Universitario in Casalina (Deruta-Perugia). Its exhibits are often made from very simple materials (such as wood, plexi-glass, wool, cardboard) and have to be touched, moved and disassembled to share geometric mental images or to communicate a mathematical idea or a theorem or an algorithm.

The collection of the *Galleria di Matematica* is continuously growing with new objects and didactic proposals, also involving the University students in producing them. A lot of collateral teaching activities are organized to promote a similar "from concrete to abstract" approach to mathematics: travelling exhibitions, shows, conferences, teacher training courses, books and teaching materials specifically made for children with special educational needs (blind, deaf, dyscalculic, having cognitive problems).

MiMa

Mathematics in the Making

Lifelong Learning Programme

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Partners

mathematikum

Sheffield Hallam University